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16 FUZZY SETS METHODOLOGIES IN ACTUARIAL SCIENCE

Richard A. Derrig
Krzysztof M. Ostaszewski

Abstract: Actuarial science encompasses all types of quantifications of risks under conditions of uncertainty for the purpose of setting insurance premiums and reserves. Traditional actuarial methodologies have been built upon probabilistic models, and are often driven by stringent regulation of the insurance business. Deregulation and global competition of the last two decades have opened the door for new methodologies, among them being fuzzy methods. Here we present the uses of fuzzy sets in areas such as: underwriting, risk classification, interest rates, ratemaking, valuation of premium and taxes. We also discuss new opportunities for expanding fuzzy sets methodologies in actuarial science.

16.1 WHY IS ACTUARIAL SCIENCE SO LATE IN JOINING THE FUZZY SCIENCE?

The area of actuarial science has been relatively late in joining the vast field of applications of fuzzy sets methodology. In order to explain why, we must first understand the nature of the actuarial problem. Actuaries are professionals who quantify risk for the purpose of deriving a price for insuring against uncertain events. One part of the work of an actuary is the evaluation of the premium collected by an insurance enterprise from its customers for the payments of future claims and benefits. Let us note that the same kind of problem applies to

the gradual funding of any future liabilities, including pension liabilities, another field of activity for actuaries. The second major part of actuarial work is the establishment of reserves. Given that the future claim payments are uncertain, it is simply impossible for the premium payment stream of cash flows of an insurance enterprise to match exactly the stream of cash flows required to pay future claims. The difference in the present value of the two must either be positive, or receive a subsidy from the insurance firm in order to be positive. This statement is simply equivalent to the insurance firm being solvent. Reserves constitute that amount, which together with future premiums, will be sufficient for the discharge of all future claim and benefit obligations. Again, this concept extends to pension funding where it is referred to as accrued liabilities.

Actuaries face uncertainty due to unpredictability of various factors influencing future premiums, claims, benefits, and investments cash flows. Traditional actuarial science (e.g., Bowers et al., 1986) assumes deterministic interest rates and a probabilistic model for the distribution of future claims. In some countries, including the United States, this has led to stringent regulation of the actual premium and reserve calculation process. Insurance is regulated at the state level in the U.S., with model national regulation written by the National Association of the Insurance Commissioners (Black and Skipper, 1994). This structure has produced a situation where the calculation of premiums and reserves for insurance is driven by legislation, sometimes specifically prescribing the methodology allowed. Indeed, until 1980, life insurance firms in the United States, for which the interest rate used in discounting future cash flows is of utmost importance, due to long term nature of their contracts, had to use one interest rate, prescribed by law effective at the inception of the contract. A change of interest rate would then require lobbying all state legislatures to change the law, not an easy task indeed. Since 1980, the interest rate effective for a given life insurance contract is a function of Moody's Investment Grade Corporate Bond Index, but still remains unchanged for the life of the contract. In addition to that, a large portion of the methodology is also prescribed by state statutes, including mortality tables. In the Commonwealth of Massachusetts premium rates for personal auto insurance are set by the Insurance Commissioner, based upon data and recommendations filed by the Auto Insurers Bureau (Derrig, 1993). These static methodologies developed in a world radically different economically from our current environment.

The Golden Age (Black and Skipper, 1994) of the U.S. insurers, the 1950s and 1960s, was characterized by nearly complete knowledge of claim related cash flows -- because of actuarial knowledge; and predictability of other

cash flows (i.e., lapses, surrenders, new business, investment returns) -- because of an economic environment providing stability of those factors. One could say that Golden Age was the "quiet before the storm". Subsequent developments (Sametz, 1987), such as:

- Unprecedented levels of inflation, and unpredictability of inflation rate;
- Unprecedented levels of volatility of financial markets, especially interest rates;
- Unprecedented deregulation, consumerism, and competition;

all leading to greater efficiency in consumer behavior, disintermediation, and change in the industry position versus other financial institutions, resulted in the insurance industry experiencing what is common in those three factors, i.e., "the unprecedented". First and foremost was the unpredictability of cash flows, or even a complete makeover of the nature of those cash flows. For example (Tullis and Polkinghorn, 1992), annuities, which have been historically a relatively unimportant part of the life insurance industry, used primarily to provide an income stream after retirement, acquire new significance as savings vehicles through the use of single and flexible premium deferred annuities, and the recent extraordinary growth of variable annuities. In 1982, total annuity reserves of U.S. life companies exceeded life insurance reserves for the first time, and by the 1990s they reached twice the level of life reserves. The popularity of annuities and other investment-related products in the United States has been aided by the provisions of the Tax Reform Act of 1986 (Babbal and Stricker, 1987). Three major milestones in the recent history of the life insurance industry (Asay, Bouyoucos and Marciano, 1993) occurred:

- In the early 1980s, the short term interest rates were at the record highs, causing massive disintermediation as policyholders fled to higher yields;
- In the middle of the 1980s, there occurred a record decline in the level of nominal interest rates, resulting in refinancing and prepayments of a large portion of insurers portfolios;
- At the end of the 1980s, insurers pursuing higher yields were often caught taking too much credit risk in their investment portfolios.

The market nature of insurance products has changed as well. Ostaszewski (1998) points out that the historical *Paul v. Virginia* Supreme Court decision of 1867, which led to state regulation of insurance, appeared to have been based on the perception of insurance as a private contract between two local parties (thus ... no interstate commerce in insurance, and no central federal government regulation). Even though there are no traded markets in insurance

products, there has been a decisive move towards competitive pricing of insurance, with mortality protection becoming nearly a commodity, and with catastrophe futures markets under development.

The traditional vision of insurance and pension valuation in the United States, before the storms described here, called for nearly deterministic calculations based on the interest rate, mortality, and other methodology prescribed by legislation. The resulting role of an actuary was limited to efficient professional calculation following the guidelines. This was in striking contrast with other English speaking nations, such as United Kingdom, Canada, or Australia, which over time developed a different role for an actuary. The concept of Appointed Actuary, as created in those countries, called upon the actuarial professional to create a model based on assumptions and methodology created by that professional, in accordance with the standards of practice, but nevertheless a creation of the actuary, which was the basis for premium and insurance. This opening for creativity was badly needed in the United States.

It was finally provided by the 1990 Amendments to the Standard Valuation Law, which require that an Appointed Actuary in the United States not only should prove that the reserves and premiums of an insurance firm are calculated properly, using a legal methodology, but also that they indeed have proper economic meaning, i.e., are sufficient to discharge company's obligations to pay claims and benefits. As a result, it is now standard that insurance firms, and indeed other financial institutions, test their long term solvency (a process often referred to as *cash flow testing*) under a large number of economic scenarios of the future, mostly interest rates scenarios, but other factors are considered, too. In addition to that, new standards of practice in Canada call for the Appointed Actuary to provide Dynamic Solvency Testing analysis of the firm. This kind of testing not only investigates the development of the company under cash flow testing scenarios, but also provides sensitivity analysis, by inquiring about the economic value of the firm under changes to various input factors, such as interest rates, mortality, policy lapsation, epidemics, etc.

We see, therefore, that the traditional actuarial analysis, although based in the probabilistic methodology, was indeed quite immune to any invasion by fuzzy sets methodology. This was especially true of the situation in the United States, given the extreme form of inflexibility written into the pre-1980 Standard Valuation Law, and even its improved pre-1990 form.

It should then come as no surprise that the early applications of fuzzy sets methodology came from European scholars, and that the decade of 1990s is marked by a sudden increase in the interest in fuzzy sets applications in actuarial science.

16.2 UNDERWRITING

The earliest work known to us directly applying fuzzy sets methodology to actuarial science was by DeWit (1982) in which he pointed out that the process of insurance underwriting, i.e., the process of selection and evaluation of risks to be insured, is indeed fraught with uncertainty which may not be properly described by probability. For example, in group insurance, even though actual pricing of the insurance product is done based on concrete data, one cannot directly apply the conclusions to any group in which participation is voluntary. Traditionally, a minimum participation rate of 75% is a requirement for issuing a group policy, but this crisp boundary is by no means a definite cure for resolving the uncertainty of the insured participants. Furthermore, the age/sex composition of the insured group is generally needed to be either stable, or reach a steady-state eventually. What constitutes "stable" is subject to the underwriter's judgement, a fuzzy approach indeed. Other factors of similar nature include description of the industry, if the group is work-related, credit rating of the policyholder, and stability of the insurance provider (a group which changes the insurance carrier every year is generally a very poor risk). Finally, there is the classical actuarial problem of *credibility* (Fuhrer, 1993) of group experience. Typically a group seeking insurance will provide an insurer with information about its claims history. These data provide one basis for premium derivation. But is it prudent to base the pricing decision on such limited data? Should industry-wide, or country-wide experience be used instead? If the claims experience of the group is better, i.e., its payments are lower, than the corresponding industry or country, the group may not welcome such generalization. The standard approach to this problem is to assign a degree of credibility, a number between 0 and 1, to the premium derived based on the group data, and one minus that number to the premium derived from the industry or country experience. The weighted average of the two is then used as the premium for the group. One cannot avoid noticing a striking similarity between this process and the fuzzy sets methodology. Karvovskii and Ostaszewski (1992, 1993) investigated development of fuzzy-based credibility measure in insurance.

The work of DeWit (1982) was followed by Erbach (1990, also see Erbach and Seah, 1993) who in 1987, together with his two colleagues, Douglas Holmes and Robert J. Purdy, working for a Canadian insurance firm, developed *Zeno*, a prototype life insurance automated underwriter using a mixture of fuzzy

and other techniques. *Zeno* was intended to do final underwriting of the majority of individual life insurance cases. The prototype was carried far enough so that it proved feasible, at which point the company turned it over to the regular systems, and it was promptly abandoned in favor of traditional human judgement. One may indeed wonder if this was a proof of any weakness of fuzzy methodology, or a signal of the world not yet ready for innovation.

In the United States one area which begs for innovation in underwriting is nonmedical issues. Some small amount individual life products are issued without gathering any medical information about the prospects, just based on data in the simple application form. Better methods of distinguishing between acceptable and not acceptable risks in this area, which *Zeno* appeared to have been able to handle, could yield significant profits. We can only wait and see if such a breakthrough will indeed happen.

Lemaire (1990) expanded on the work of DeWit (1982) by suggesting fuzzy logic methodology for insurance underwriting in general, as well as fuzzy calculation of insurance premiums and reserves. The underwriting methodology underwent further refinement in the work of Young (1993), which published a specific algorithm for group health underwriting, utilizing fuzziness of rules such as mentioned above. Horgby, et al. (1997) introduced fuzzy inference rules by generalized modus ponens as a means of underwriting mortality coverage for applicants with diabetes mellitus. Twenty seven medically related factors are represented as fuzzy input parameters to a fuzzy controller scheme with a center of area defuzzifier to extract a crisp premium surcharge.

16.3 USING FUZZY ACTUARIAL PRESENT VALUES AND FUZZY ARITHMETIC

Lemaire's work (1990) included calculations of present values under fuzzy interest rates and fuzzy factors influencing future cash flows being discounted. Calculations of actuarial present values, i.e., expected values of present values of future random payments, are at the very core of actuarial science, especially when applied to life insurance and annuities. The pioneering work in this area, based on the standard rules of fuzzy arithmetic (see, e.g., Zimmerman, 1991) was done by Buckley (1986, 1987) and was an extension of the classical mathematics of finance. Calzi (1990) expanded on Buckley's ideas. Although traditional actuarial science as applied to life insurance assumed the interest rate to be constant and given for the entire duration of the contract, there is a wide recognition of the uncertainty of interest rates. In his discussion of the time value of money, Trowbridge (1989) put it very well: "The inexperienced actuary

may tend to take an assumption about the time value of money as a given, and devote little or no attention to the appropriateness of the interest rate assumed. As he gains knowledge and experience, however, the actuary learns to differentiate between gross interest and net, before and after tax, nominal, effective, and real rates of interest, and internal rate of return. He gains a knowledge of the yield curve, the relationship between interest rates for different maturity periods. He recognizes that any specific interest rate has a basic component for time preference, and additional components for the possibility of default and the expectation of inflation. He knows that interest rate changes can affect assets and liabilities differently." Indeed, the last challenge identified by Trowbridge may be a unique field for deep investigations utilizing fuzzy set methodology. When valuing assets, one can usually determine their market values, or use market interest rates reasonably closely related to the risk of default of the entity providing asset cash flows. But on the liabilities side, do we expect the actuary to provide for the risk of default of the firm he or she works for? How would the clients of the insurance firm view it, if they learned that the firm is considering a possibility of defaulting on the promises of claim and benefit payments made to them? But on the other hand, is it reasonable to exclude one's default risk, while including it in assets of firms of similar risk profile that the firm holds among its investments? The fuzziness lurking beneath this problem calls for an innovative approach indeed.

Yet another statement of actuarial principles brings about the inherent vagueness of certain considerations of financial natures. Dicke *et al.* (1991) write "Actuaries are often called upon to place a value on future contingent cash flows related to the operations of a financial security system. Because the actuarial value is, in general, a random variable, it may be preferable to state the conditions under which the actuarial value may be expected to fall within a given range." This is a direct endorsement of probabilistic models. Such probabilistic models of interest rates in the insurance firm model have been proposed by Panjer and Bellhouse (1980), Frees (1990), Dufresne (1992), and others. But the immense complexity of stochastic models brings about calls for simplicity, at least in presentation of the analysis performed. One of the key challenges of modern insurance theory lies in the area of asset/liability management. This refers to the process of protecting the company, and its profits, in the environment of changing interest rates. One tool of asset/liability management is the concept of *duration*, defined as the logarithmic derivative of the price of a financial instrument (e.g., insurance firm asset or liability). A simple model of an insurance firm compares duration of assets and duration of

liabilities, usually calling for the two to be equal, or close to each other, to match the sensitivity of assets and liabilities to changes in interest rates. Unfortunately, duration-based analysis has some pitfalls. The main one is represented by the so called *Short Straddle Model* of an insurance firm which proclaims that a typical firm with assets and liabilities of similar duration will lose its economic value under any change of interest rates (Babbel and Stricker, 1987). A much better view of the company situation is obtained if one can see how the economic value changes with changes in uncertain interest rates. Ostaszewski (1993) points out that traditional fuzzy finance models call for the uncertainty of interest rates to be represented by Figure 1 below:

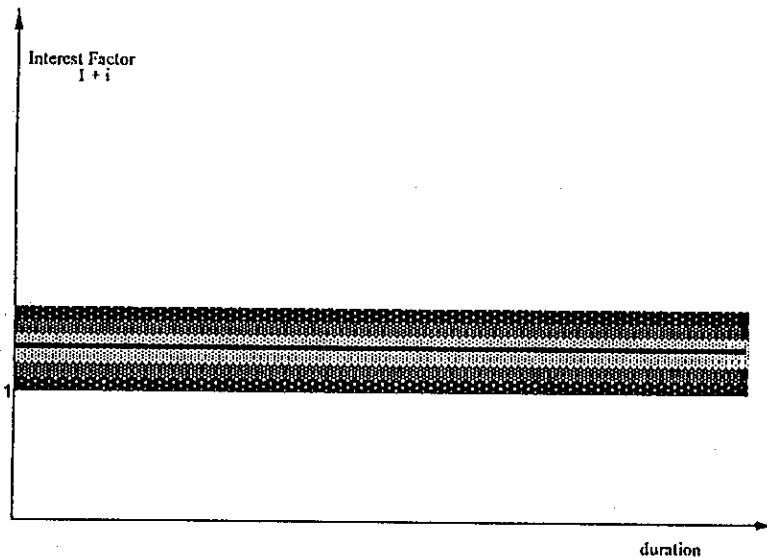


Figure 1 Uncertainty of Interest Rate with No Duration Variation

Figure 1, however, is not a realistic picture of interest rates. The near term interest rates are generally less uncertain than the long term returns which typically results in an upward sloping yield curve. Furthermore, one can receive a premium for undertaking the risk of a longer term investment. Thus a better picture of fuzzy interest rates may be the one presented below in Figure 2. The resulting "fuzzy yield curve" can then be used to calculate fuzzy present values,

and can provide a very good representation of sensitivity to changes in interest rate levels.

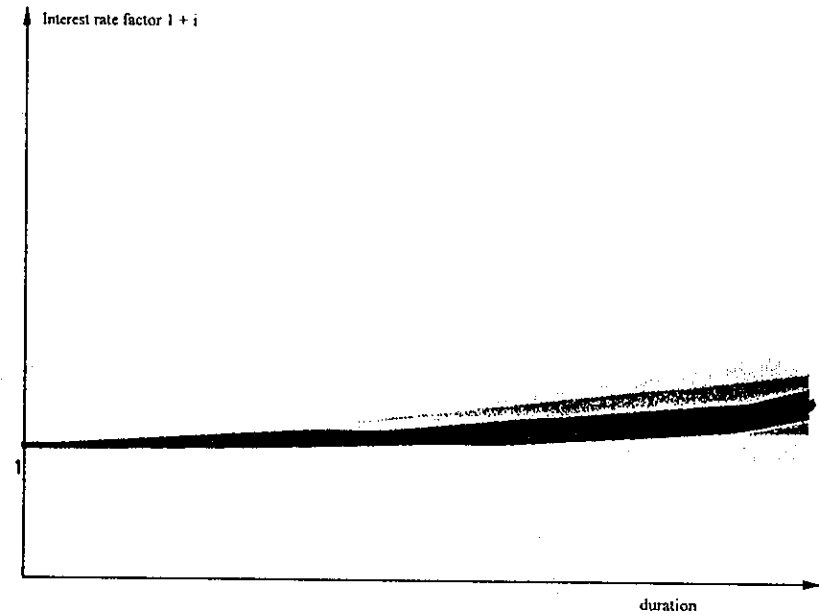


Figure 2 Uncertainty of Interest Rate with Duration Variation

We should properly note that the vast field of investment analysis, which is of natural interest to actuarial scientists, has been effectively "infiltrated" by fuzzy sets methodology, used mostly to create expert systems for the purpose of security selection and portfolio design. An example of such work is given by Wong, Wang, Goh and Quak (1992), but we should note that successful expert systems tend to remain proprietary, due to tremendous monetary incentives involved.

16.4 RISK AND CLAIM CLASSIFICATION

Ostaszewski (1993) pointed out that insurance risk classification often has to resort to rather vague and uncertain methods, or methods which are excessively precise -- as in a case of a person who may fail to classify as a preferred risk for

life insurance application because of having body weight exceeding the stated limit by half a pound (this was also noted by Lemaire, 1990). Kandel (1982), writing from a different perspective, says: "In a very fundamental way, the intimate relation between the theory of fuzzy sets and the theory of pattern recognition and classification rests on the fact that most real-world classes are fuzzy in nature."

Ebanks, Karwowski, and Ostaszewski (1992) use measures of fuzziness to classify risks. In many situations, we do know in advance what characteristics a preferred risk possesses. Any applicant can be compared, in terms of measurements featured in the characteristics, to the "ideal" preferred risk, and then a membership degree can be assigned to each measurement. This produces a feature vector of fuzzy measurements describing the individual. By measuring the fuzziness of that individual as a preferred risk, we can determine a new classification.

Derrig and Ostaszewski (1995) use fuzzy clustering for risk and claim classification. They use the fuzzy c-means algorithm as discussed by Bezdek (1981). Let us illustrate this with a very simple example, originating from Lemaire (1990), and partly from Ostaszewski (1993). Suppose that four prospective insureds are defined by four characteristics, height, gender, weight, and resting pulse, and initially classified by sex. Given the following data:

Person 1: Height 175 cm, Gender 0 (male), Weight 92 kG, Resting Pulse 110;

Person 2: Height 185 cm, Gender 0 (male), Weight 75 kG, Resting Pulse 75;

Person 3: Height 160 cm, Gender 1 (female), Weight 55 kG, Resting Pulse 72;

Person 4: Height 150 cm, Gender 1 (female), Weight 90 kG, Resting Pulse 100.

Persons 1 and 2 have a degree of membership in Cluster 1 of 1.00, and in Cluster 2 of 0.00, with persons 3 and 4 having exactly the opposite situation. We should note that the example given is intentionally exaggerated, but it does refer to the rather fundamental question of how to determine the nature of clusters of similar risks in insurance. If significantly different risks are insured under one rate, good risks tend to either underinsure, or leave in pursuit of a better deal. This process is commonly referred to as *adverse selection* (*Casualty Actuarial Society, 1990, p.35*). The end result is that the insurance firm is not only losing good customers, but it ends up insuring a group with significantly higher expected future claims. On the other hand, too fine of a classification results in increased costs, while crisp schemes produce situations such as a person 1 cm too short for a preferred rate ending up paying a significantly higher premium, because of this nearly nonexistent distinction from preferred risks. Ostaszewski (1993) points out that lack of actuarially fair classification is economically equivalent to price discrimination in favor of high risk prospects.

When the fuzzy c-means algorithm is applied to the above group of four risks, clustering gradually changes away from gender-based to a fuzzy

partitioning combining inputs from all four factors (gender, weight, height, and resting pulse) used to measure risk. This is illustrated in the following figures.

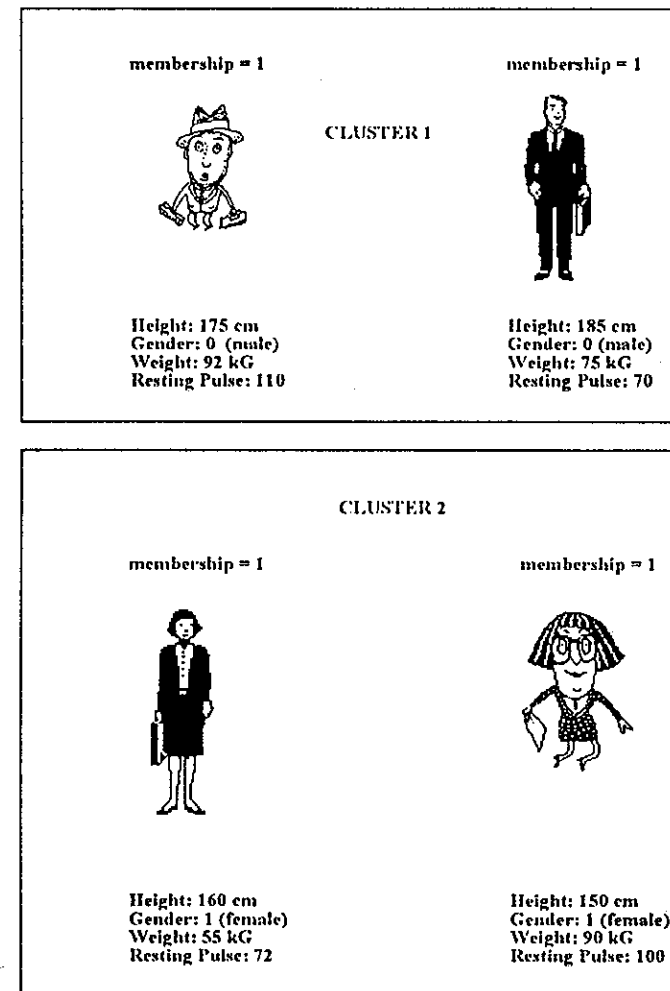


Figure 3 Initial Partition

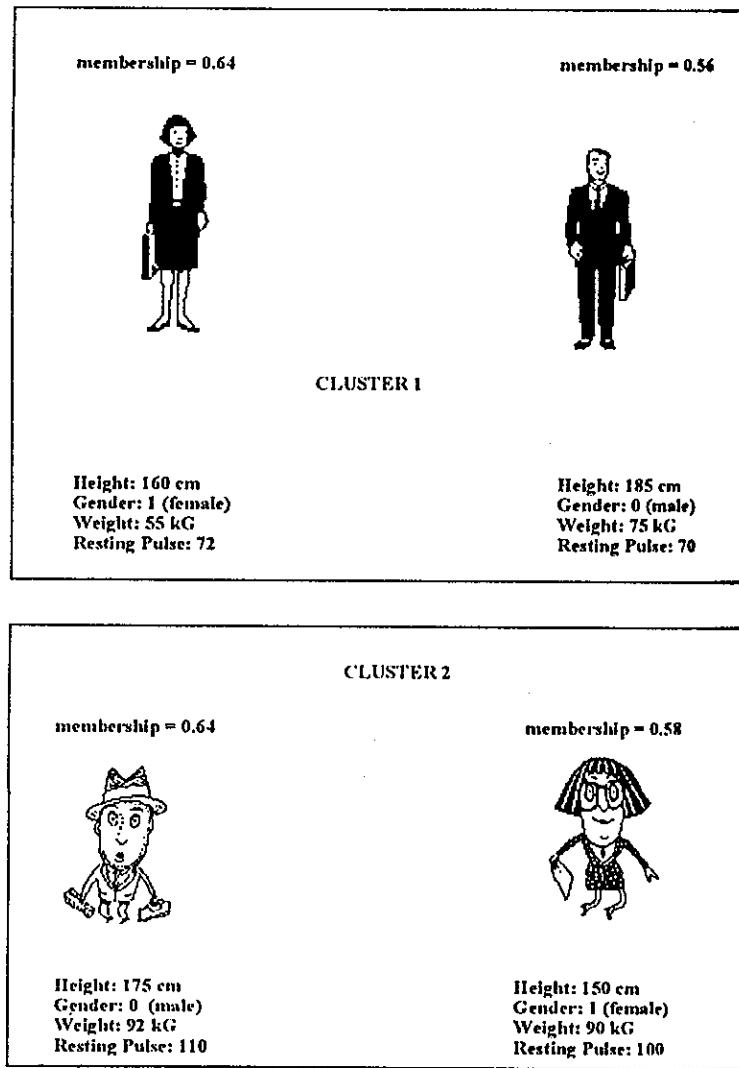


Figure 4 Final Iteration Partition

Derrig and Ostaszewski (1995) applied the fuzzy c-means algorithm to the problem of automobile territory rating in Massachusetts. As Conger (1987) describes "In Massachusetts, the past ten years have witnessed the evolution of an increasingly sophisticated system of methodologies for determining the definitions of rating territories for private passenger automobile insurance. In contrast to territory schemes in other states, which tend to group geographically contiguous towns, these Massachusetts methodologies have had as their goal the grouping of towns with similar expected losses per exposure, regardless of the geographic contiguity or non-contiguity of the grouped towns."

The methodology used in Massachusetts for arriving at town groupings results in pure premium indices for each of the 360 towns (or, more precisely, 350 towns and ten areas into which Boston is divided for automobile rating purposes). The indices, which are numbers relatively close to 1 (either greater than 1 or smaller, the indices represent expected losses in relation to those of the entire state expected losses) are then ordered and territories are created by analyzing such ordering. Since frequent switches from one territory to another are undesirable, numerous restrictions on moving towns from one territory to another exist. Also, capping is used, which restricts the maximum price movement of any town. Such difficulties in clustering warranted an investigation of fuzzy clustering. Resulting fuzzy clusters are much more flexible, as a town belonging partially to two territories could in the final assignment belong to the one of them which is more appropriate because of the regulatory limitations. It should be noted that although stability of territory assignment is desirable and convenient, the system of clustering towns into territories should meet the standard criterion for risk classification -- it should be responsive, to loss control (or lack thereof). Towns have an incentive to reduce their relative loss costs by maintaining their roads, law enforcement, safety engineering, and law enforcement, if those actions bring about lower premiums. If the system is not responsive, or slow to respond, the incentives are lost.

Derrig and Ostaszewski (1995) applied the fuzzy c-means algorithm for the 350 non-Boston towns, as the ten Boston towns are traditionally separated. The pure premium indices were calculated for the following coverages for all 350 towns: Bodily Injury Liability; Personal Injury Protection, Property Damage Liability, Collision, Comprehensive, and Combined. The data for the 1993 indices were used. The initial clustering was the actual 1993 territory assignment, i.e., there were 16 non-Boston territories. In addition to the above calculation, they also performed a calculation adding two more coordinates for

each town -- its geographical coordinates (latitude and longitude) divided by the coordinates of Boston (the division is performed to adjust the scale for the numbers to other coordinates, which are all close to 1). By performing the algorithm on these coordinates they increased the chance of arriving at clusters which are not only actuarially similar, but also relatively close geographically. Recall that other states in the United States do use geographical proximity as an important factor in determining rating territories. The results of the work of Derrig and Ostaszewski (1995) were quite revealing. The traditional regulatory problem in Massachusetts has been the shifting of certain towns from one territory to another, and back. Fuzzy clustering showed that for those towns their membership in territories is indeed best described as fuzzy, and a more flexible regulatory approach may be needed for future decisions.

It should be noted that the work of Derrig and Ostaszewski (1995) contained some pioneering insight into use of fuzzy sets methodology to detect fraudulent claims in property-liability insurance. Cox (1995) devised fuzzy parameters to compare individual medical provider behavior to a peer group for the purpose of detecting "anomalous" behavior (better known as fraud and build-up) in health insurance. Insurance fraud is a very significant problem of the industry, especially in the presence of weak economic disincentives. Social insurance programs, such as Workers' Compensation, Medicare, and Social Security in the United States, have been often victimized by unscrupulous individuals submitting fraudulent claims. This detection has been traditionally done by special investigative units (SIU) in claim processing, (IRC, 1997) and requires significant time and money outlays, sometimes with only modest returns (Derrig and Weisberg, 1998). It should be frankly admitted that pursuit of every claim exaggerated by a few dollars does not make economic sense. Fuzzy clustering allows for distinguishing between grossly fraudulent claims and the so called claim build-up. The insurer is then free to investigate those suspicious claims which show the greatest threat of fraud. Detection of insurance fraud is still a relatively recent application of fuzzy sets methodology, and we believe that there are many promising opportunities ahead for it.

16.5 PROPERTY/CASUALTY INSURANCE PRICING

As we have already pointed out, one part of the job of an actuary is the determination of the cost of an insurance products. What is the cost of the insurance product? It is simply the cost of future covered claims, expenses and taxes. As simple as we attempt to make this definition, its practical application may not turn out to be so, given the quite fuzzy nature of the concepts of

"covered" (what about insurance fraud and build-up?) claims, expenses, and taxes. Furthermore, once the cost is determined, we proceed to the price of an insurance product, which is the expected cost plus an expected profit margin. Since both the cost and profit margin are uncertain, there has to be some model of that uncertainty. It has traditionally been probabilistic. Derrig (1990) discusses the development of property/casualty insurance pricing in the United States.

The greatest challenge for property/casualty insurance lies in forecasting of claim costs, as well as in providing for fair profit and taxes. Cummins and Derrig (1993, 1997) proposed a fuzzy model for property/casualty insurance pricing. They began with claim costs trends. As we pointed out above, premium estimates are based upon expected claim costs for the period of coverage. Expected claim costs are based upon historical data, but must be developed to ultimate costs (i.e., we must provide for all already incurred costs of claims, and costs which may appear in the further processing of the claims), and must be trended to the policy period (i.e., must provide for expected changes in claim costs in the future caused, for example, by inflation). We see that the trends studied here are fuzzy, due to factors such as:

- Selection of historical data and data periods used in estimations;
- Forecasting models used, curve-fitting or econometric;
- Curves to be used, linear, exponential, and others;
- Statistical measures of fit, accuracy and bias;
- Reasonability of results, and
- Possible incompatibility of fit and reasonability.

Cummins and Derrig (1993) studied claim cost trends, and compared existing forecasting methods with respect to their forecasting accuracy, bias, and reasonability. Their main conclusion was that forecast methods that are nearly as accurate and unbiased may not produce expected claim costs that are nearly the same. They suggested assigning a membership degree to a method for its accuracy, bias and reasonableness separately. They then derived a composite fuzzy inference measure of the accuracy, bias, and reasonableness of a forecasting method. This produced much greater insight into the value of various methods than the commonly used methods comparisons of regression R-Squares or the preference of a company actuary.

Cummins and Derrig (1997) also provide examples of calculations of fuzzy insurance premiums for property/casualty insurance. They note that the premium calculation faces a first level uncertainty due to:

- cash flow magnitudes;
- cash flow patterns;
- risk free interest rates;
- risk adjustments, and
- tax rates.

These uncertainties are traditionally handled through probabilistic models and actuaries' judgement. But there is also second level uncertainty due to:

- historical data quality
- development methods
- trend or forecasting methods
- expense allocations
- surplus allocations
- capital market modeling, and
- insurance markets

By using, fuzzy parameters: losses, risk-free rate, risk-adjustment, and crisp parameters: flow patterns, tax rates, Cummins and Derrig (1997) derived fuzzy insurance premiums as generalizations of the crisp net present value pricing model (Myers and Cohn (1987), Derrig (1990). One interesting comment the authors have about their results is that the answers turned out to be "fuzzier" than they expected, indicating a degree of uncertainty which may not have been accounted for in traditional pricing methods.

Although presenting a prospective client with a fuzzy premium is not realistic, we should note that a fuzzy price may be a very valuable tool in assessing the range of premiums which needs to be considered, as well as in informing the management of the uncertainty of the premium calculation process.

16.6 FUZZY TAXES

Income taxes have a major effect on product pricing and insurance-investment portfolio management (Derrig, 1994). Derrig and Ostaszewski (1997) develop applications of fuzzy sets methodology to the management of the tax liability of a property/casualty insurance company. Myer's Theorem (1984) says that the risk-adjusted present value of the tax liability on investment income from a risky investment portfolio held by a corporation is

$$PV(T\tilde{r}_A) = \frac{T r_F}{1 + r_F}$$

where r_A is the rate of return on the risky portfolio and r_F is the risk-free rate of return. The present value of the tax liability is independent of the investment strategy, and determined solely by the effective tax rate and the risk free rate. Derrig and Ostaszewski (1997) use fuzzy sets techniques to evaluate the effective tax rate as a fuzzy number, by considering fuzzy investment returns on a portfolio of government bonds, stocks, and within a context of a liability portfolio providing a tax shield.

For example, the effect of liability tax shield on the effective tax rates with fuzzy investment returns and liability shield are presented below in the Figure 5. We see that this work provides a tool for estimation of the effects of liabilities on taxes.

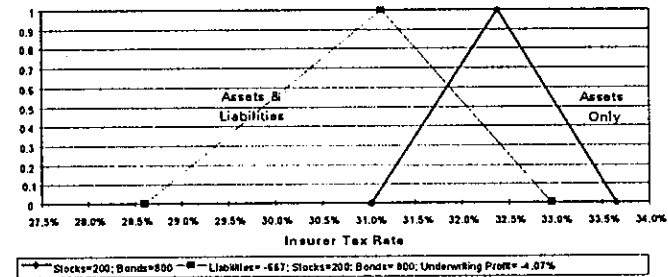


Figure 5 Fuzzy Investment Tax Rates: Effect of Liability Tax Shield

Figure 6 provides a picture of the effective tax rate under varying asset portfolio compositions. Cummins and Grace (1994) determined that property/casualty insurers in the United States perceive a yield advantage for longer maturity tax exempt bonds, implying the existence of a portfolio with an effective tax rate lower than 35 percent (the current corporate income tax rate in the United States). This can be justified only by a tax clientele effect -- a marginal buyer with a marginal tax rate of less than the insurers' 35% less their 5.1% minimum proration, alternative minimum tax rate, and capital gains

income. But this perception of insurers is merely just that, a perception, and further studies are needed to determine if it is grounded in reality. The analysis of the effective tax rate under varying portfolio compositions, as in the Figure 6, suggests that small allocation shifts between bonds and stocks may have little effect on the expected tax rate.

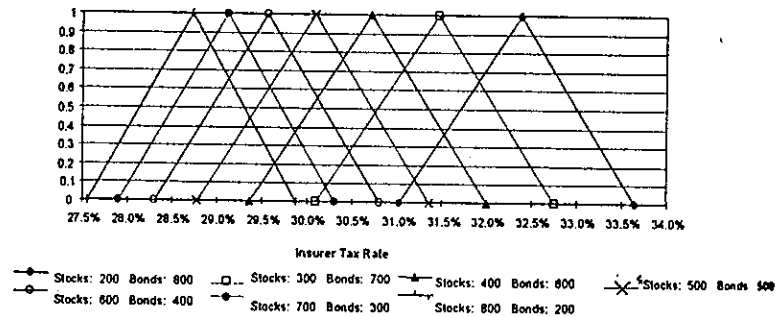


Figure 6 Fuzzy Investment Tax Rates with Selected Asset Mixes

Derrig and Ostaszewski (1995) also determine that the beta one company (i.e., a company with the same degree of risk as the whole market) has a fuzzier variation around the expected as the leverage of liability to surplus increases. This implies increasing uncertainty with increased leverage, a fact intuitively perceived by management, but now possibly quantified with fuzzy methodology, as in Figure 7.

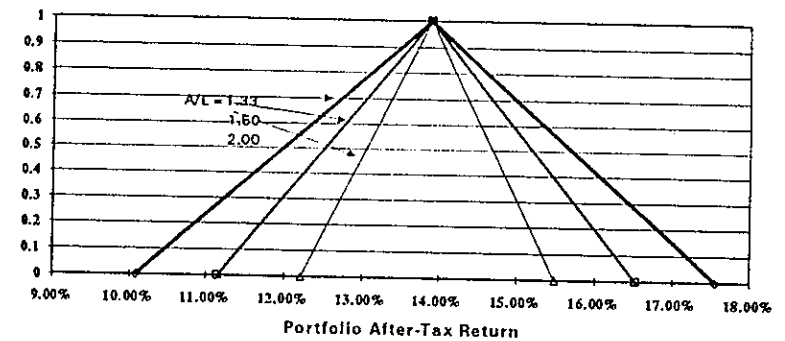


Figure 7 Fuzzy After-Tax Returns Effect of Levered Investments

16.7 THE FUTURE OF FUZZY SETS METHODS IN ACTUARIAL SCIENCE

Ostaszewski (1993) provided a broad overview of possible applications of fuzzy sets methodology in actuarial science. Recent developments do indicate that expert systems in the areas of underwriting and fraud detection hold the greatest promise in immediate applications. Indeed, such applications may be under development among the most competitive insurers.

But in the longer perspective, fuzzy sets can provide a great opportunity in risk classification, and in providing a new, fresh perspective on integrated management of an insurance firm. Some recent developments are also promising in combining fuzzy sets methodology with intervals of possibilities (Babad and Berliner, 1994), Kohonen's Self-Organizing Feature Map (Brockett, Xia, and Derrig, 1998), and neural networks. Operational control of premium price changes via fuzzy logic would begin adapting fuzzy controller methods in industrial processes to financial decisions. For example, Young (1996) codified several common actuarial concepts as fuzzy parameters in a rate changing controller. Prior rate changes, actual to expected ratios, change of business, and cancellations all combine in a modus ponens inference to produce a rate change indication for a line of business. Horgby (1998) discusses a similar fuzzy inference system for life insurance classification. Basic mortality classes are supplemented by fuzzy parameters such as overweight, high blood pressure and elevated cholesterol that combine to determine a high risk surcharge. Even

though the entrance of fuzzy sets into actuarial science has been somewhat delayed, especially in the United States, we will probably see more progress in this area, especially in view of increasing competitiveness and globalization of the insurance industry.

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